

**ESTIMATE A KERNEL REGRESSION FOR FUNCTIONAL  
INDEPENDENT DATA IN SEMI-PARAMETRIC  
MODELS THROUGH THE APPLICATION  
OF THE LEAVE-ONE-OUT TECHNIQUE  
FOR BANDWIDTH SELECTION**

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SUMMARY

In this manuscript, we address the estimation of the conditional regression function for a scalar response variable  $Y$  conditioned on a Hilbertian random variable  $X$ , under the assumption that the observations exhibit a single-index structure. The paper showcases the pointwise almost complete convergence, along with the corresponding rate, of the kernel estimator proposed for this particular model. To illustrate the applicability of our findings, we demonstrate how our outcomes manifest in the context of simulation studies involving functional data. This is achieved through the employment of the leave-one-out (LOO) technique within cross-validation for the purpose of selecting the bandwidth parameter governing the smoothness of the estimated function. We also analyze a spectrometric data set.

*Keywords and phrases:* Functional random variable, kernel estimator, single functional index model, functional regression.

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