

LOWER SEMICONTINUITY OF FIRST PASSAGE TIME IN STOCHASTIC VOLATILITY MODELS AND ITS IMPACT ON BARRIER OPTION PRICING

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SUMMARY

This paper investigates the lower semicontinuity of the first passage time (τ) in stochastic volatility models and its implications for barrier option pricing. The first passage time plays a critical role in financial mathematics, particularly in the valuation of barrier options, where the payoff depends on whether the underlying asset crosses a predetermined barrier before expiration. We rigorously prove that τ is lower semicontinuous, ensuring stability in pricing models and preventing abrupt changes in option values due to small perturbations in asset paths or volatility trajectories. To complement the theoretical results, we conduct targeted Monte Carlo simulations to validate Theorem 1 and quantify empirical violation rates under different stochastic volatility bands. The numerical findings confirm that small variations in initial conditions or volatility produce low violation rates and stable barrier option prices, with an observed asymmetry, upward perturbations are more likely to trigger earlier barrier hits than downward ones. These results reinforce the robustness of barrier option pricing models under stochastic volatility and highlight the practical importance of the lower semicontinuity property for reliable risk management and hedging strategies.

Keywords and phrases: First Passage Time, Lower Semicontinuity, Stochastic Volatility Models, Barrier Options, Monte Carlo Simulations, Financial Mathematics.

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