

## Some Applications of Mixed Gamma Representations

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### Abstract

Mixed gamma representations are given for representing the difference of two chi-square random variables and for order statistics for a random sample of size  $n$  from an exponential distribution. The distribution for the first representation can be expressed as a finite mixture of chi-square random variables with mixture probabilities that are recognizable from World Series-type problems. An implication is that the simple representation can be used to create exact probability plots for plotting estimated variance effects in fractional factorial designs with normal errors rather than the traditional, but approximate, normal probability plots based on log transformations.

The second representation shows that the  $k^{th}$  order statistic,  $X_{k;n}$ , can be represented as the ratio of two independent random variables. In the ratio, the numerator is a gamma with shape parameter  $k$  and the denominator is a size biased, shifted  $(k - 1)$ -fold convolution of uniform distributions. Since the numerator is the asymptotic limit of  $X_{k;n}$  as  $n$  goes to infinity, the denominator completely captures the error in using the gamma as the approximate distribution. This relationship is used to investigate the asymptotic behavior of the relative error in this gamma approximation.

**Keywords:** Threshold representation, probability plotting, exponential order statistics, error in approximation, World Series problem.

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